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Appearance (Cesia): Construction of Scales by Means of Spinning Disks

The name cesia has been proposed for the aspect of appearance that deals with the sensations aroused by differences in the spatial distribution of light. In a previous article, a system of cesias was presented. This included a notation system and a solid with three dimensions or variables in which the sensations were organized. The aim of this article is to report the development of scales of cesia, constructed with the aid of rotatory disks holding percentages of materials producing the stimuli for the five primary sensations of cesia. These primary sensations are: transparency, specular reflection, translucence, diffuse reflection, and absorption (black). Different geometries of observation were needed to evaluate the scales. Numerically, they were built using a power function between the magnitudes of the sensations and the magnitudes of the stimuli. This research refutes the assumptions that the geometric attributes of appearance cannot be organized in a coordinate system and that synthetic stimuli cannot be produced for these attributes just as they are produced for color by the mixture of three lights. © 1994 John Wiley & Sons, Inc.

Key words: appearance, cesia, geometric attributes of appearance, spatial distribution of light, permeability, absorption, diffusivity, transparency, translucence, specular reflection, diffuse reflection, matte, psychophysical scales, spinning disks.

INTRODUCTION

The term appearance alludes, in a broad sense, to a whole series of visual aspects in which color, texture, glossiness, translucence, transparency, and even sometimes the shape and size of objects are included. In this context, color is usually defined as the aspect of appearance that results from differences in the spectral distribution of light, the aspect to which we refer by saying that something is, e.g., red, green, blue, yellow, white, or black. Beyond this, there exists a whole field, which refers to those conditions producing differences in the spatial distribution of light, causing something to be seen as transparent, translucent, specular, glossy, matte, etc.

For these visual sensations, no generic term has broad acceptance. Hunter's refers to these aspects as geometric attributes of appearance, also including texture under this heading. Green–Armytage discriminates these aspects from texture and labels them as quality of surfaces. César Jannello coined the term cesia, which also excludes texture and seems appropriate for phenomena that are independent of color, though related to it. Jannello arbitrarily derived the word cesia from his own name, César; however, it could be said to have some relationship with the Latin verb cedere (cede, transfer, yield). In this sense, cesia could be interpreted as the way light is transferred (in regard to its spatial distribution) and reaches an observer. Claudio Guerri, in a verbal comment during the 1st Argentine Color Conference, also remarked that Jannello made an association with caesura, which in poetry means a pause, a break in the flow of sound. In this sense, cesia would refer, if not to breaks, to changes or alterations in the flow of light. Thus, the term cesia is employed here for the visual sensations that depend on the spatial distribution of light, in the same way as the term color is used for the sensations that depend on the spectral distribution of light.

Cesia, like color, is not an intrinsic property or attribute of materials and surfaces. The physical properties of a material in regard to the transmission, absorption, reflection, and scattering of light, the conditions of illumination (intensity and direction of light), and the view-
ing angle, all contribute to the production of a certain sensation of ceisia; the same object can appear to have different cesias if one or another of those conditions are varied. Figure 1.1 shows a group of dried hop flowers, which appear translucent with back lighting and opaque with front lighting. A piece of glass looks transparent if it is observed perpendicularly to its surface, and behaves more like a mirror when looked at from an angle whose deviation from the perpendicular is close to 90° (Fig. 1.2). A porcelain statuette looks matte when it is illuminated by a diffuse light at relatively low intensity, and appears glossy when it is illuminated by a spotlight at relatively high intensity (Fig. 1.3). The glass of a window changes from transparent to specular (mirrorlike) as the light, from being more intense outside the room (natural light during the day) becomes more intense inside the room (artificial light at night) (Fig. 1.4).

It is necessary to bear in mind the difference between the stimulus, the physical variable that can be objectively measured by instruments, and the sensation, the psychological variable that is subjective (or can be considered as intersubjective if we are referring to an average observer). The two variables do not generally correlate in a linear fashion, that is, a certain increment of the stimulus does not always represent an equivalent increment of the sensation. Fechner’s psychophysical law, which applies in some cases, establishes that the sensation varies proportionally to the logarithm of the stimulus. Stanley Stevens found that the sensation (subjective or apparent magnitude) grows in relation to a power of the magnitude of the stimulus. The exponent is different for each kind of sensorial perception and is defined by experimental methods in each case.

In producing scales of cesia one must ensure, as in the case of color, that the intervals are perceptually equal. Thus, if we establish a scale with ten steps, step 5 must be perceived as the halfway sensation between step 1 and step 10, and the interval between steps 1 and 2 must be seen as equal to the interval between steps 2 and 3, 3 and 4, and so on.

METHOD

Five sensations of cesia—considered as the primary sensations—were used to generate the scales. At this first stage it was decided to keep the question of color separate. Thus, only achromatic or colorless primary cesias—that is, without selectivity in regard to the wavelength of the light reflected or transmitted by the samples—were chosen. The primary sensations are: clear transparence (colorless), specular reflection (mirrorlike appearance, colorless), translucence (white), diffuse reflection (matte white), and complete absorption (black).

In order to produce the stimuli that make up these five primary sensations, the following ideal standards are considered:

* The suffix “ion,” as in reflection and absorption, are generally used here to speak of sensations, while the suffix “ance,” as in reflectance,

Transparence: a perfect transmitting material, with coefficient 1 (100%) of regular transmittance. Air meets the conditions for such an ideal.

Specular reflection: a perfect mirror, with coefficient 1 (100%) of regular reflectance. A surface of aluminium evaporated onto glass is the standard that most closely approaches this ideal.

Translucence: a perfectly diffuse and totally transmitting material, with coefficient 1 (100%) of diffuse transmittance. There is no material that sufficiently approaches this physical ideal. Such a material would be one that: (a) placed at the open top of a black velvet-lined box and illuminated from outside looks black (this means that it permits light to pass through toward the interior of the box without producing any reflection at its surface); and (b) placed against a spotlight and seen from the other side looks as a uniform white surface (this means that it permits light to pass through but it scatters light in its totality).

Diffuse reflection: a perfect matte white, with coefficient 1 (100%) of diffuse reflectance. The material that most closely approaches this ideal is a surface of pure barium sulphate powder.

Absorption: a perfect black, with coefficient 1 (100%) of absorbance. The device that produces a stimulus approaching this ideal is a black velvet-lined box with only a small aperture to look inside.

In the manufacture of the disks, the following materials were used, all of them achromatic or colorless: air for transparence, metallized polyester 125 μm thick for specular reflection, film polyester 210 μm thick for translucence, white matte cardboard for diffuse reflection, and black matte cardboard for absorption.

These materials—except for air, which in clear, limpid atmospheric conditions and at short distances has a transmittance of 100%—merely approach the ideal stimulus for each case. The values in Table 1 are measurements of real samples that can practically be considered as the best ones obtainable; in the case of transparence it is possible to use air, which comes up to the ideal standard.

According to the model of cesia presented in a previous article, eight scales of five steps each were constructed (Fig. 2). The scales, kept within the field of achromatic or colorless sensations and defined by their opposite poles, are:

1. matte white—black
2. specular—black
3. translucent—black
4. transparent—black
5. specular—matte white
6. transparent—translucent
7. specular—transparent
8. matte white—translucent

Scales of absorption
Scales of diffusivity
Scales of luminosity
Scales of regularity
Scales of permeability
Scales of opacity

Transmittance, and absorbance, are used to refer to measurements of stimuli.
FIG. 1. The sensation of cesia varies with the illumination or the angle of view. The same objects, under different illuminating conditions or seen at different angles, appear to have different cesias. (1.1) The petals look translucent with back lighting and opaque with front lighting. (Reproduced from P. Brodatz, Textures: A Photographic Album for Artists and Designers, Dover, New York, 1966). (1.2) The glass looks specular or transparent, depending on the angle of view. (1.3) The statuette looks matte under a diffuse illumination and glossy under a more concentrated one. (1.4) The window looks transparent during the day, when the most intense source of light is outside the room, and turns specular as night advances, when the artificial light inside the room becomes the most intense source.
TABLE 1. Measurements of real samples.

<table>
<thead>
<tr>
<th></th>
<th>Reflectance (%)</th>
<th>Transmittance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regular</td>
<td>Diffuse</td>
</tr>
<tr>
<td>White</td>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>Black</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Mirror</td>
<td>95</td>
<td>1</td>
</tr>
<tr>
<td>Translucent</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Transparent</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The variation of sensation in each scale can be expressed in one direction (for instance, sensation of absorption) or in its opposite (sensation of luminosity). Going from one pole to the other in the scale, one of the sensations increases while the opposite decreases.

The steps in each scale are expressed by means of coefficients (0–0.25–0.50–0.75–1) or by means of percentages (0%–25%–50%–75%–100%). These coefficients or percentages do not refer to the stimuli; they are numbers that express magnitudes of sensations. This means that when we read 0.50 or 50% it is not to be assumed that the opposite stimuli in the scale are present in equal or half parts, but that the sensation experienced is such that it is perceived as the midpoint between the two primary sensations at the extremes of the scale.

Maxwell’s method of spinning disks for additive mixture of two or more color stimuli was adapted for building scales of cesias. With the speed of rotation, the separate colors on a Maxwell disk fuse and a new color is seen. Similarly a scale from, e.g., specular reflection (mirrorlike appearance) to absorption (black) would be constructed with disks on which the relative size of the mirror and black segments could be varied.

Geometric Dispositions for the Observation of the Disks

Particular geometries of observation, special conditions of illumination, and disposition of elements were necessary in order to evaluate each kind of scale.

1. The white–black scale is a typical scale of opaque grays. The spinning disks were illuminated with diffuse light from 45° and observed at 90° against a medium gray background (Fig. 3.1). This scale was used as a point of departure in view of the possibility of comparing it with the existing scales of grays (Munsell specially) and also because the other three scales of absorption/luminosity (those that have black as one of their poles) were constructed in reference to this one.

2. For the specular–black scale, a matte white surface was placed in front of the disks and the reflection produced by the disks was observed. The aim of this was to match the sensations produced by the already constructed scale of grays. (Fig. 3.2).

3. For the translucent–black scale, a white background was illuminated behind the disks and was observed at 90° through the disks. The purpose of this was to match the sensations of the scale of grays (Fig. 3.3).

4. For the transparent–black scale, the same procedure was followed; a white background was illuminated

---

**FIG. 2.** The solid of cesias with the five primary sensations and the three kinds of variation. The eight scales developed are placed along the eight edges. In addition to these, any other kind of scale can be constructed, either by moving over the outer surface or going to the inside of the solid.
behind the disks and was observed at 90° through the disks. The objective was again to match the scale of grays (Fig. 3.3).

For the two scales of diffusivity/regularity, a surface with a uniform visual pattern was illuminated; then, the goal was to produce variations, with perceptually equal intervals, of the distinctness of the pattern.

5. For the specular–white scale, the surface was placed in front of the disks for it to be reflected by them (Fig. 3.4).

6. For the transparent–translucent scale, the pattern was placed behind the disks and observed through them (Fig. 3.5).

7. For the scale of permeability/opacity specular–transparent, a pattern of parallel lines was placed behind the disks, and a surface with the same pattern, but positioned with the lines perpendicular to the former, was placed in front of the disks (Fig. 3.6). I first looked for the middle point of the scale, that in which a checked pattern with the same intensity in both directions was observed. From this point to each of the two extremes, the intermediate sensations were determined.

8. The scale of permeability/opacity white–translucent is particularly difficult to determine because here, as in the previous scale, the subjective quantities of light seen by transmission and reflection must be evaluated, but, due to the diffusiveness, this determination cannot be made with the aid of linear patterns. Instead, a surface with a saturated color far enough from

FIG. 3. Geometries adopted for the evaluation of the different scales of cesia.
white in regard to its luminosity (an intense blue) was placed behind the disks. The illumination was directed both to the surface behind and to the front of the disks. With this arrangement, the extreme of the scale having the translucent sample appeared blue while the white extreme looked white. The scale was evaluated as a gradation from hue to white (Fig. 3.7).

All these arrangements were made to build the scales. Now, in order to show the scales already built in a way that they can be differentiated from one another, it is necessary in some cases to change some elements in the arrangements so far described. The changes can be noticed by comparing the drawings in Fig. 3 with the photographs in Fig. 4.

Thus, as already explained, the remaining three scales of absorption/luminosity (specular–black, translucent–black, and transparent–black) were built placing a white surface behind or in front of the disks with the goal of evaluating them in comparison with the white–black scale. In such conditions the four scales look identical; in order to differentiate them when showing them, it is necessary to change the kind of surface seen by reflection or transmission.

Then, for the specular–black scale, the fact that it is seen by reflection and that such a reflection is regular (unlike the white–black scale in which the reflection is diffuse) is evinced by placing a surface with a visual pattern in front of the disks and a nonpatterned background. The disks will reflect the pattern (Fig. 4.2).

For the translucent–black scale, the fact that it is seen by transmission and that such transmission is diffuse is evinced by placing a pattern behind the disks. The light is seen to come from the rear, while the pattern completely disappears (it becomes diffuse) in the sectors seen through the disks (Fig. 4.3).

For the transparent–black scale, the fact that it is seen by transmission and that such transmission is regular is evinced by placing, instead of the white background used to build the scale, a background with a uniform pattern. This way, we can realize that what is seen through the disks is the radiation coming from such a background (Fig. 4.4).

Figure 4 shows photographs of the eight scales of cesia made by disks under rotation. From 4.1–4.4, the four scales of absorption/luminosity are displayed; Figures 4.5 and 4.6 display the two scales of diffusivity/regularity; Figures 4.7 and 4.8 show the two scales of permeability/opacity.

In the Appendix, a possible method to calculate the stimuli that make up perceptually equi-spaced scales is described.

OTHER SCALES

Up to this point, the scales developed are only the eight that appear at the edges of the solid of cias. Two more scales exist that can also be included among the basic ones. They are the transparent–white (Fig. 5.1) and specular–translucent scales, which appear in the diagonals of the upper surface of the solid. In addition to these, many other scales are possible. An infinite number of them can be marked by tracing lines going between intermediate points of the solid (such as semi-transparent, semi-reflecting, semi-matte, etc.) or by linking any other pair of points.

Examples of scales of cesia can be observed in every-
larly to its surface and tilting it progressively up to a position deviated somehow less than $90^\circ$ from the perpendicular.

Scales also arise by variation in the intensity of the incident light. Thus, if we look through a window while the night is falling we will have a transparent–black scale (provided that the sun is the only source of light present). If the glass of such a window is frosted, then we will have a translucent–black scale. If the light falling upon a white surface gradually diminishes, a white–black scale is observed; if the same happens upon a mirror, a specular–black scale can be seen.

CONCLUSIONS

There are some assumptions that I intend to reverse through the approach described here and in previous studies. Hunter affirms that, unlike colors, which can be identified by means of three-dimensional coordinate systems, the geometric attributes such as gloss, diffusion, translucence, and others can hardly be organized and cannot be defined by any system of dimensions. The system of cesias, built on the basis of three variables, as well as the scales that can be generated through it, prove that such a task is possible. Even more, though I will not try to demonstrate or argue this point now, I want to advance the hypothesis that the different cesias in which a color can appear can be explained in the general terms of the trichromatic theory.

Although it is true, as Lozano points out, that a perfect mirror does not possess color and that gloss masks color (in the shining sector of a surface we can hardly see the color of the surface because we see the reflection of the source of light), it is also true that there are tinged mirrors and that sometimes the gloss possesses hue. This is also explainable in terms of the system of cesias and the trichromatic theory. This will be the case of surfaces with spectral selectivity for the component of specular reflection (Fig. 6).

According to Hunter, “it is not possible to produce an equivalent gloss stimulus synthetically (as color stimuli are matched by mixtures of three standard lights).” This article is intended to demonstrate that the visual appearance of any object can be matched by means of spinning disks, starting from the mixture of two or more primary stimuli.

Perspectives and Applications

At present, designers (industrial and graphic designers, architects, etc.) can benefit from the use of systems for the description and measurement of color; systems that allow them to specify a certain tone with great accuracy. The measurement can be done by visual comparison with standard colors in current color atlases such as Munsell's or by means of measurement employing specific instruments.
cesias, that is, to the extremes of transparence, translucence, specular reflection, diffuse reflection, and absorption, traversing all the intermediate steps. Even the primary sensations—except for the absorption (black)—may appear having spectral selectivity; we can have a colored transparence (as in a transparent filter), a colored translucence, a colored specular reflection (as in a tinged mirror), and, of course, a colored diffuse reflection (as in a matte color). The different cesias in which a color may appear can be adequately explained in terms of the trichromatic theory by referring the variables or dimensions of cesia to each primary component (red, green, or blue).

The only feature that the solid of cesias shares with the traditional color solids is the line on which the scale of absorption/luminosity runs from a diffuse reflecting surface to black. The variable called absorption (or its opposite, luminosity) in cesia is similar to the variable called value, lightness, darkness, or luminosity in color. The difference is that, in cesia, this variable is also applied to transparent, specular, and translucent sensations, which are not taken into account in the traditional systems of color. If for each opaque color, to reach all the possible cesias, an individual solid of cesias is given, then colors differing in hue or saturation but having equal value will be located in equivalent points of those solids of cesias. Thus, for example, all the tones of the Munsell atlas (matte finish), including the scale of grays, are located in the line of the solid of cesias going from opaque matte to absorbent (or in equivalent lines, if we allow a different solid of cesias for each different hue and saturation). The position of these colors varies only according to their value or luminosity.

With these facts as a starting point, and by using spinning disks to produce optical mixture, any sensation of cesia associated with any color can be matched.

ACKNOWLEDGMENTS

Special thanks are due to Roberto Daniel Lozano (Consultora Color Internacional) for the measurement of the samples with the spectrophotometer The Color Machine (Byk-Gardner Inc.) with his software “Calidad.” I owe gratitude also for the comments of one anonymous reviewer and for discussions with María L. Fago de Mattiello regarding psychophysical scales.

APPENDIX: NUMERIC CONSTRUCTION OF THE SCALES

This appendix describes the procedure employed in this occasion to build the scales. This is not the only way to accomplish this task but just an example of how it can be done. Furthermore, the apparatus, material samples, measurements, and subjective evaluations could be improved to make the calculation of the stimuli that make up the scales more exact.

As a point of departure, a relation of a power function
between sensation and stimulus was adopted, as expressed by the law of Stevens:  
\[
\text{sensation} = k \cdot \text{stimulus}^\beta. \tag{1}
\]

In our case, the problem is to calculate the stimuli corresponding to a scale of subjective magnitudes previously fixed. Then,

\[
\text{stimulus} = \sqrt[\beta]{\frac{\text{sensation}}{k}}. \tag{2}
\]

The value 0.5 was first adopted for \( \beta \) in the white–black scale. This number gives a series of reflectance stimuli, which closely concur with the reflectances in Munsell’s scale of grays.  

Our 5-step scale of grays was built with this exponent (placed as a root in this case). As has been said, all the scales of absorption/luminosity were built by comparison to this one. Thus, for each one of the specular–black, translucent–black, and transparent–black scales, a series of five disks were produced holding the same proportions as the scale of grays, that is, using the same value of \( \beta \). When they were visually judged under rotation, they proved to be correct.

For the other scales, different values of \( \beta \) were tested with a method of successive approximations.

According to the observations made, uniform scales were obtained with the following values of \( \beta \):

- **Scales of absorption/luminosity**
  - White–black \( \beta = 0.5 \)
  - Specular–black \( \beta = 0.5 \)
  - Translucent–black \( \beta = 0.5 \)
  - Transparent–black \( \beta = 0.5 \)

- **Scales of diffusivity/regularity**
  - Specular–white \( \beta = 1 \)
  - Transparent–translucent \( \beta = 0.7 \)

- **Scales of permeability/opacity**
  - Specular–transparent \( \beta = 1.6 \)
  - White–translucent \( \beta = 1 \)

Resorting to Stevens’ formula, and adding now the stimulus in relation to the threshold (in our case this threshold is the minimal value of reflectance or transmittance of the actual samples), we have:

\[
\text{sensation} = k \cdot (\text{stimulus} - \text{threshold})^\beta, \tag{3}
\]

\[
\text{stimulus} = \sqrt[\beta]{\frac{\text{sensation}}{k}} + \text{threshold}, \tag{4}
\]

\[
k = \frac{\text{sensation}}{(\text{stimulus} - \text{threshold})^\beta}. \tag{5}
\]

The sensations are fixed in a 5-step scale: 0, 25, 50, 75, 100%. The ideal stimuli, as we have seen, have either 0 or 100% of reflectance or transmittance, according to the stimulus in question. The obtainable real stimuli cover the following ranges:

<table>
<thead>
<tr>
<th>Range (%)</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>90–2% of diffuse</td>
<td>white–black</td>
</tr>
<tr>
<td>reflectance</td>
<td></td>
</tr>
<tr>
<td>95–1% of regular</td>
<td>specular–black</td>
</tr>
<tr>
<td>reflectance</td>
<td></td>
</tr>
<tr>
<td>85–0% of diffuse</td>
<td>translucent–black</td>
</tr>
<tr>
<td>transmittance</td>
<td></td>
</tr>
<tr>
<td>100–0% of regular</td>
<td>transparent–black</td>
</tr>
<tr>
<td>transmittance</td>
<td></td>
</tr>
<tr>
<td>95–1% of regular</td>
<td>specular–white</td>
</tr>
<tr>
<td>reflectance</td>
<td></td>
</tr>
<tr>
<td>100–0% of regular</td>
<td>transparent–translucent</td>
</tr>
<tr>
<td>transmittance</td>
<td></td>
</tr>
<tr>
<td>95–0% of regular</td>
<td>specular–transparent</td>
</tr>
<tr>
<td>reflectance</td>
<td></td>
</tr>
<tr>
<td>90–12% of diffuse</td>
<td>white–translucent</td>
</tr>
<tr>
<td>reflectance</td>
<td></td>
</tr>
</tbody>
</table>

In regard to the ideal stimuli, the constant \( k \) gets the following values:

\[
k_1 = 10 \quad \text{when} \quad \beta = 0.5,
\]

\[
k_2 = 3.981 \quad \text{when} \quad \beta = 0.7,
\]

\[
k_3 = 1 \quad \text{when} \quad \beta = 1,
\]

\[
k_4 = 0.063 \quad \text{when} \quad \beta = 1.6.
\]

When calculated with respect to the real stimuli, the constant \( k \) holds different values, as shown in the upper-right corner of every scale in Table II.

Table II shows different kinds of values corresponding to each type of scale.

Column \( A \) gives values to one of the two sensations in the extremes of the scale; column \( B \) lists the values of the opposite sensation. As sensation in column \( A \) decreases, that in column \( B \) grows.

Column \( A_1 \) gives the ideal stimuli in percentages of reflectance or transmittance, whether it be diffuse or regular. These values are obtained with formula (2), in which \( \beta \) and the constant \( k \) vary according to the kind of scale, and the sensation is taken from column \( A \).

Column \( A_2 \) gives the corresponding disk sectors measured in degrees.

\[
A_2 = \frac{A_1 \cdot 360^\circ}{100}.
\]

Column \( B_1 \) gives the percentages of the other sector of the disk. Its values are obtained by subtracting the values in column \( A_1 \) from 100.

\[
B_1 = 100 - A_1.
\]

Column \( B_2 \) gives the corresponding disk sectors measured in degrees.
TABLE II. Numeric construction of the scales (the explanations are given in the text).

Scales of absorption/luminosity

White-black (90 to 2% diffuse reflectance) \( \beta = 0.5 \quad k_i = 10 \quad k_r = 10.660 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal white</th>
<th>Ideal black</th>
<th>Wht ctr</th>
<th>Blk ctr</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (lum)</td>
<td>( B ) (abs)</td>
<td>( A_1 ) (refl)</td>
<td>( A_2 ) (deg)</td>
<td>( B_1 ) (%)</td>
<td>( B_2 ) (deg)</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>56.25</td>
<td>202.5</td>
<td>43.75</td>
<td>157.5</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>25</td>
<td>90</td>
<td>75</td>
<td>270</td>
</tr>
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<td>25</td>
<td>75</td>
<td>6.25</td>
<td>22.5</td>
<td>93.75</td>
<td>337.5</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

Specular-black (95 to 1% regular reflectance) \( \beta = 0.5 \quad k_i = 10 \quad k_r = 10.314 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal specular</th>
<th>Ideal black</th>
<th>Spc ctr</th>
<th>Blk ctr</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (lum)</td>
<td>( B ) (abs)</td>
<td>( A_1 ) (refl)</td>
<td>( A_2 ) (deg)</td>
<td>( B_1 ) (%)</td>
<td>( B_2 ) (deg)</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>56.25</td>
<td>202.5</td>
<td>43.75</td>
<td>157.5</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>25</td>
<td>90</td>
<td>75</td>
<td>270</td>
</tr>
<tr>
<td>25</td>
<td>75</td>
<td>6.25</td>
<td>22.5</td>
<td>93.75</td>
<td>337.5</td>
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<tr>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

Translucent-black (85 to 0% diffuse transmittance) \( \beta = 0.5 \quad k_i = 10 \quad k_r = 10.846 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal transluc.</th>
<th>Ideal black</th>
<th>Trl ctr</th>
<th>Blk ctr</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (lum)</td>
<td>( B ) (abs)</td>
<td>( A_1 ) (trsm)</td>
<td>( A_2 ) (deg)</td>
<td>( B_1 ) (%)</td>
<td>( B_2 ) (deg)</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
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<td>25</td>
<td>56.25</td>
<td>202.5</td>
<td>43.75</td>
<td>157.5</td>
</tr>
<tr>
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<td>50</td>
<td>25</td>
<td>90</td>
<td>75</td>
<td>270</td>
</tr>
<tr>
<td>25</td>
<td>75</td>
<td>6.25</td>
<td>22.5</td>
<td>93.75</td>
<td>337.5</td>
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<tr>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

Transparent-black (100 to 0% regular transmittance) \( \beta = 0.5 \quad k_i = 10 \quad k_r = 10 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal transpar.</th>
<th>Ideal black</th>
<th>Trp ctr</th>
<th>Blk ctr</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (lum)</td>
<td>( B ) (abs)</td>
<td>( A_1 ) (trsm)</td>
<td>( A_2 ) (deg)</td>
<td>( B_1 ) (%)</td>
<td>( B_2 ) (deg)</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>56.25</td>
<td>202.5</td>
<td>43.75</td>
<td>157.5</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>25</td>
<td>90</td>
<td>75</td>
<td>270</td>
</tr>
<tr>
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<td>75</td>
<td>6.25</td>
<td>22.5</td>
<td>93.75</td>
<td>337.5</td>
</tr>
<tr>
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<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

Scales of diffusivity/regularity

Specular-white (95 to 1% regular reflectance) \( \beta = 1 \quad k_i = 1 \quad k_r = 1.0638 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal specular</th>
<th>Ideal white</th>
<th>Spc ctr</th>
<th>Wht ctr</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (reg)</td>
<td>( B ) (dif)</td>
<td>( A_1 ) (refl)</td>
<td>( A_2 ) (deg)</td>
<td>( B_1 ) (%)</td>
<td>( B_2 ) (deg)</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
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<td>0</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>75</td>
<td>270</td>
<td>25</td>
<td>90</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>50</td>
<td>180</td>
<td>50</td>
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<td>90</td>
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<tr>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

Transparent-translucent (100 to 0% regular transmittance) \( \beta = 0.7 \quad k_i = 3.981 \quad k_r = 6.309 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal transpar.</th>
<th>Ideal transluc.</th>
<th>Trp ctr</th>
<th>Trl ctr</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (reg)</td>
<td>( B ) (dif)</td>
<td>( A_1 ) (trsm)</td>
<td>( A_2 ) (deg)</td>
<td>( B_1 ) (%)</td>
<td>( B_2 ) (deg)</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>66.3</td>
<td>239</td>
<td>33.7</td>
<td>121</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>37.15</td>
<td>134</td>
<td>62.85</td>
<td>226</td>
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<tr>
<td>25</td>
<td>75</td>
<td>13.8</td>
<td>50</td>
<td>86.2</td>
<td>310</td>
</tr>
<tr>
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<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>
### TABLE II. (Continued)

**Scales of permeability/opacity**

Specular–transparent (95 to 0% regular reflectance)  \( \beta = 1.6 \quad k_d = 0.063 \quad k_r = 0.0685 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal specular</th>
<th>Ideal transpar.</th>
<th>Spc ctrl</th>
<th>Trp ctrl</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ( \text{(opc)} )</td>
<td>B ( \text{(pm)} )</td>
<td>( A_1 ) ( \text{(refl)} )</td>
<td>( A_2 ) ( \text{(deg)} )</td>
<td>( B_1 ) ( \text{(%)} )</td>
<td>( B_2 ) ( \text{(deg)} )</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>83.54</td>
<td>301</td>
<td>16.46</td>
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</tr>
<tr>
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<td>64.84</td>
<td>233</td>
<td>35.16</td>
<td>127</td>
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<tr>
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<td>75</td>
<td>42.04</td>
<td>151</td>
<td>57.96</td>
<td>209</td>
</tr>
<tr>
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<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

White–translucent (90 to 12% diffuse reflectance)  \( \beta = 1 \quad k_d = 1 \quad k_r = 1.282 \)

<table>
<thead>
<tr>
<th>Sensation</th>
<th>Ideal white</th>
<th>Ideal translc.</th>
<th>Wht ctrl</th>
<th>Trl ctrl</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ( \text{(opc)} )</td>
<td>B ( \text{(pm)} )</td>
<td>( A_1 ) ( \text{(refl)} )</td>
<td>( A_2 ) ( \text{(deg)} )</td>
<td>( B_1 ) ( \text{(%)} )</td>
<td>( B_2 ) ( \text{(deg)} )</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>100</td>
<td>360</td>
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<td>0</td>
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<tr>
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<td>90</td>
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<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>360</td>
</tr>
</tbody>
</table>

\[ B_2 = \frac{B_1 \cdot 360^\circ}{100} \quad \text{or} \quad B_2 = 360^\circ - A_2. \]

Column \( A_3 \) gives the contributions to the stimuli made by one of the real samples of the scale in terms of their reflectance \( (R) \) or transmittance \( (T) \), whether it be diffuse or regular.

\[ A_3 = \frac{A_1 \cdot R \text{ or } T \text{ real}}{100}. \]

Column \( B_3 \) gives the contribution to the same stimulus made by the other real sample in the scale.

\[ B_3 = \frac{B_1 \cdot R \text{ or } T \text{ real}}{100}. \]

The total magnitude of the stimulus appears in column \( C \), whose values are obtained by adding the contributions made by each of the two samples of the scale.

\[ C = A_3 + B_3. \]

It can be verified that the values in column \( C \) concur with the values obtained with formula (4) applied to each kind of scale in particular. Values in this column follow the same increment as those in column \( A_1 \), but covering a minor range given by the stimuli produced by the real samples.

As each of these scales covers a different range according to the stimuli produced by the real samples, it can be easily noticed that the perceptual intervals are equally spaced inside each particular scale, but they are slightly different from one scale to another.

In order to use these scales while keeping the homogeneity of the perceptual intervals throughout all the solid of ciasias, the eight borders of the solid should be scaled according to the values given in columns \( A_1 \) of Table II. Within this ideal scaling, the intervals produced by the real scales can be distributed proportionally (columns \( C \) of Table II). These real scales will generally not reach the vertices of the solid. In this way, the length of the intervals marked in the solid will be proportional to the perceptual difference among the intervals of each scale.

3. Ref. 1, p. 4.
9. Ref. 1, p. 247; Ref. 8, p. 170.
11. Ref. 8, p. 376, provides a table with the luminous reflectance of the different Munsell values.
15. Green-Armytage (ref. 2) arrived independently at the same conclusion and also proposed an order system to organize what he calls "quality of surfaces."


17. Ref. 1, p. 65.

18. Ref. 1, pp. 218–47.

19. See, for instance, Faber Birren (ed.), A Grammar of Color (A Basic Treatise on the Color System of Albert H. Munsell), Van Nostrand Reinhold, New York, 1969, pp. 46–70. Principles like these are based upon the organization of colors in an order system, which allows the selection of bundles of harmonic colors by following certain paths.

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